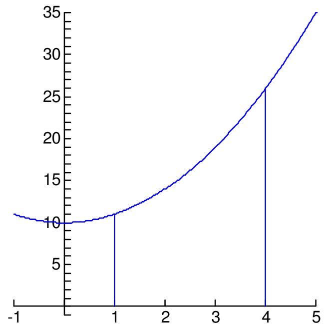
4-2 (Day 2): Riemann Sums
Calculus AB

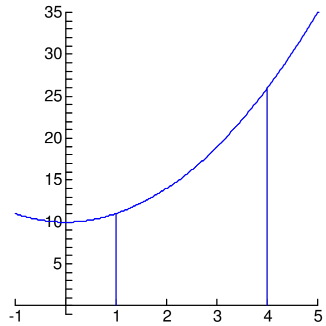
Name ______________________

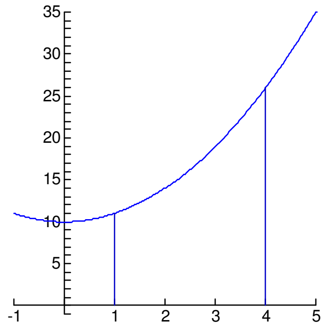
Introduction to Riemann Sums

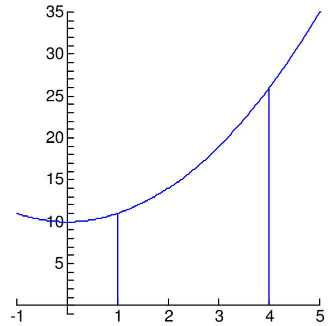
Suppose the velocity of a moving object is given by $v(t) = t^2 + 10$ where $v(t)$ is in feet per minute and t is in minutes. The area of the region represents ft/min times minutes which calculates the distance traveled by the object from t = 1 to t = 4. In each diagram below, we will find the approximate area by dividing the region into rectangles. The width of each rectangle will be Δt and each rectangle's length will be $v(t)$. The sum of the areas of these rectangles is called a Riemann sum.

Example 1) Approximately how far did the object travel between t = 1and t = 4 minutes.









If we increase the number of rectangles (the width of each gets smaller) we can obtain even better estimates.

As n increases, L_n and R_n get closer to each other and to the actual area. In reality, the exact area is the $\lim_{n\to\infty} R_n$ or $\lim_{n\to\infty} L_n$ or $\lim_{n\to\infty} M_n$ or $\lim_{n\to\infty}$ of a sum based on ***ANY*** point in the subinterval.

Riemann Sum

A Riemann sum, S_n, for function f on the interval [a, b] is a sum of the form

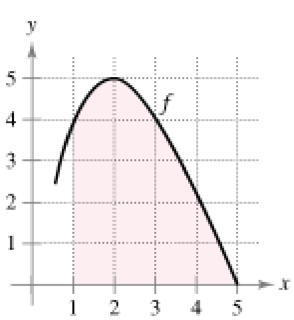
$$S_n = \sum_{k=1}^{n} f(c_k)\Delta x$$

Where the interval [a, b] is partitioned into n subintervals of widths Δx (so $\Delta x = \frac{b-a}{n}$)

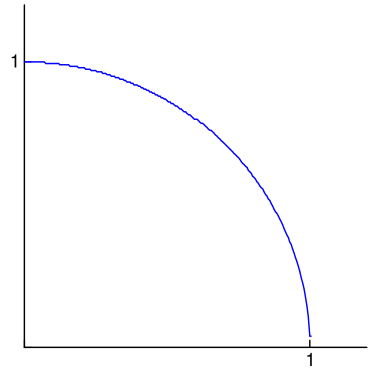
and the numbers c_k are sample points, one in each subinterval (left hand, right hand, midpoint, or any other point in the subinterval).

- If each sample point is picked so that $f(c)$ is the lowest in its respective subinterval, then each rectangle has an area that is less than the actual area. In this case the Riemann sum is called a lower sum.
- An upper sum is a Riemann sum with each sample point taken where $f(c)$ is the highest in its respective subinterval.
- A midpoint sum is formed by choosing each sample point at the midpoint of the respective subinterval.
- An upper sum is an upper bound for the area of the region and a lower sum is a lower bound. The actual area must be somewhere between the two.

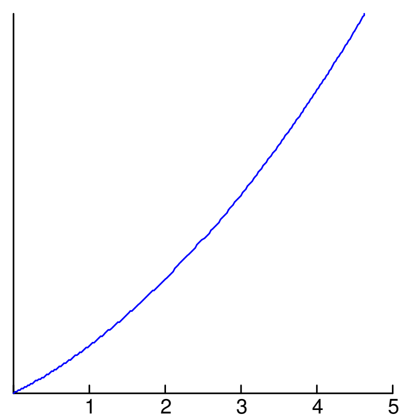
Example 2) Bound the area of the shaded region by approximating the upper and lower sums. Use rectangles of width 1.



Example 3) Use a right Riemann sum and a left Riemann sum to approximate the area of the region bounded by the x and y axis and $f(x)=\sqrt{1-x^2}$ using 5 subintervals.



Example 4) Use a midpoint Riemann sum with $n=4$ to approximate the area of the region bounded by $f(x)=x^2+4x$ and the x-axis on the interval [0, 4].



Example 5) Use the table of values and a right and a left Riemann sum to approximate the area of the region bounded by $f(x)$ and the x-axis on the interval [0, 15].

x	0	3	6	9	12	15
$f(x)$	50	48	44	36	24	8